Root Locus Design

MEM 355 Performance Enhancement of Dynamical Systems

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Outline

The root locus design method is an iterative, graphical procedure for selecting and tuning compensators.

- Basic Compensator Types
- The Root Locus Design process
- Example: BWR Pressure Control
- Hard Control Problems – a brief introduction

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Basic Compensators

$G_c(s)$	Name	Effect on Ultimate State Error	Effect on Stability
K	P (uncompensated)		
$K\frac{s+\alpha}{s}$	PI	Improves	Degrades
$K\frac{s^2+\alpha_1 s+\alpha_0}{s}$	PID	Improves	Improves somewhat
$K\frac{s+\alpha}{s+\beta}, \alpha > \beta$	lag	Improves somewhat	Degrades somewhat
$K\frac{s+\alpha}{s+\beta}, \alpha < \beta$	lead	Degrades somewhat	Improves somewhat
$K(s+\alpha)$	Rate feedback (PD)	Degrades	Improves
$K\frac{s^2+2\rho_1\omega_1 s+\omega_1^2}{s^2+2\rho_2\omega_2 s+\omega_2^2}$	Notch		Neutralizes plant resonance

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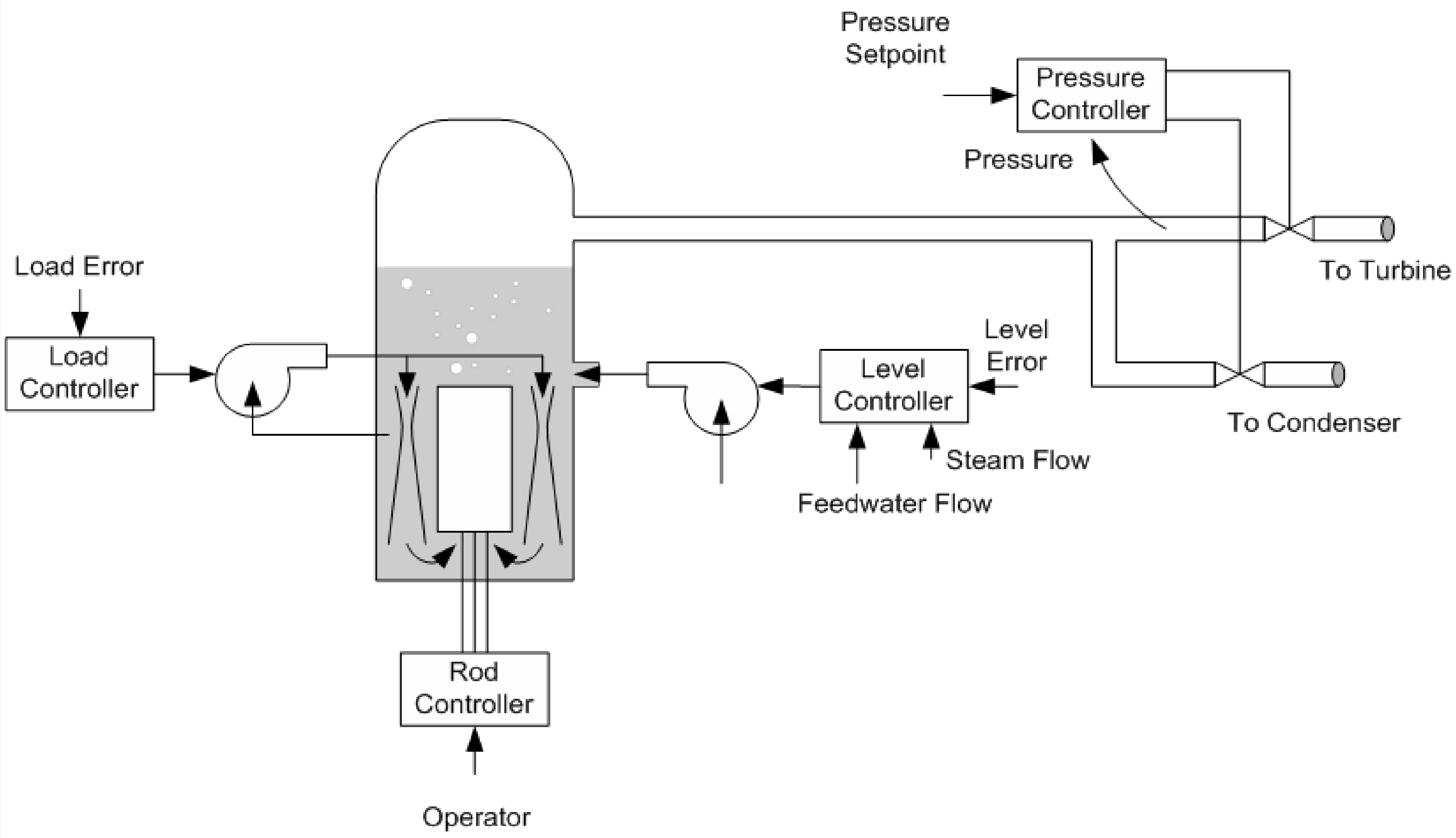
Procedure

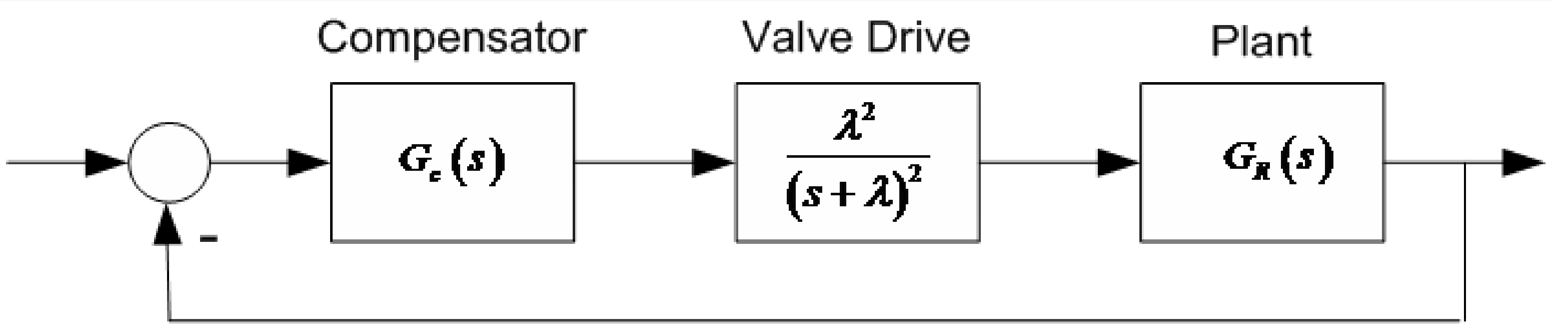
1) Uncompensated system (proportional)
   - Root locus
   - Ultimate state error/step response
2) Compensated system
   - Choose/modify compensator
   - Root locus
   - Ultimate state error/step
3) Repeat step 2)

Example: BWR Pressure Controller



BWR Pressure Control Model

Pressure command



Pressure at valve input

$$G_R(s) = \gamma \underbrace{\frac{(s - z_r)}{(s - p_r)}}_{\text{Reactor Pipe Dipole}} \underbrace{\frac{(s - z_P)}{(s - p_P)}}_{\text{Pipe Dipole}} \underbrace{\left[\frac{s^2 + 2\xi_1 \nu_1 s + \nu_1^2}{s^2 + 2\rho_1 \omega_1 s + \omega_1^2}\right] \cdots\cdots \left[\frac{s^2 + 2\xi_N \nu_N s + \nu_N^2}{s^2 + 2\rho_N \omega_N s + \omega_N^2}\right]}_{\text{Acoustics (first N harmonics)}}$$

BWR Transfer Functions

Valve | Reactor | Pipe | Acoustics 1st harmonic

$$G_p(s) = 15\left(\frac{25}{(s+5)^2}\right)\left(\frac{s+0.05}{s+0.1}\right)\left(\frac{s+0.025}{s+4}\right)\left(\frac{s^2+2(0.05)12s+144}{s^2+2(0.25)8s+64}\right)$$

$$\text{PI } G_c(s) = K\frac{s+2}{s}$$

$$\text{PI plus Notch } G_c(s) = K\frac{s+2}{s}\,\frac{s^2+2(0.15)8s+64}{(s+8)^2}$$

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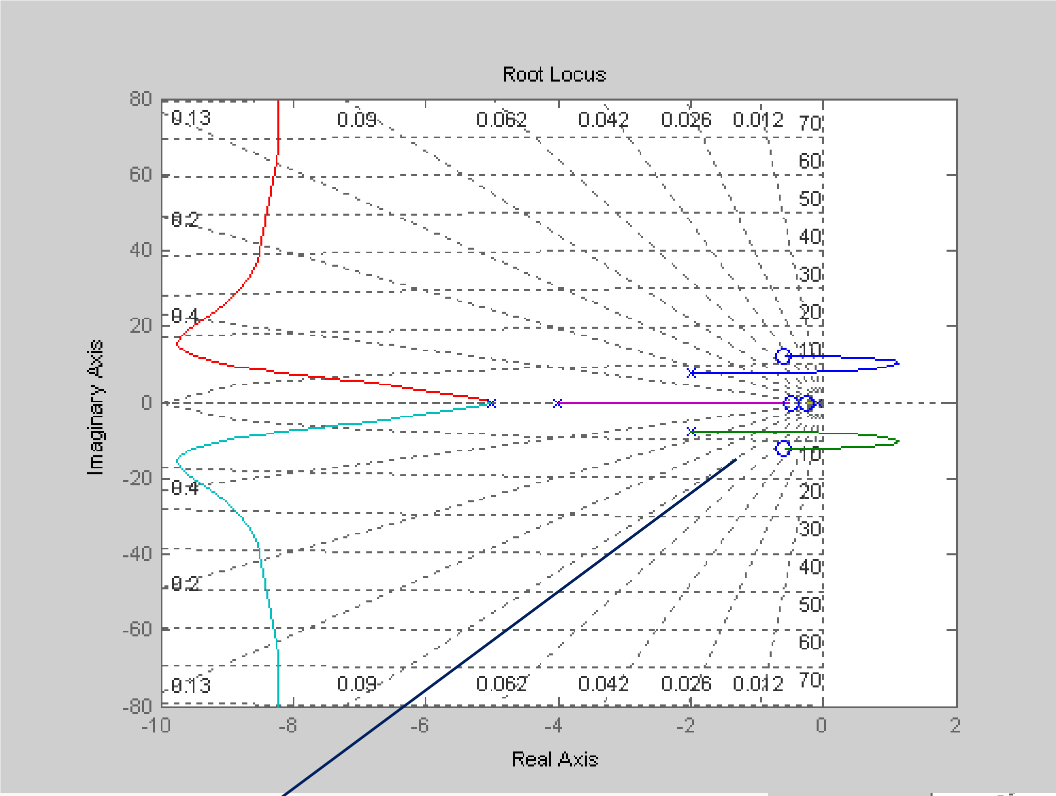
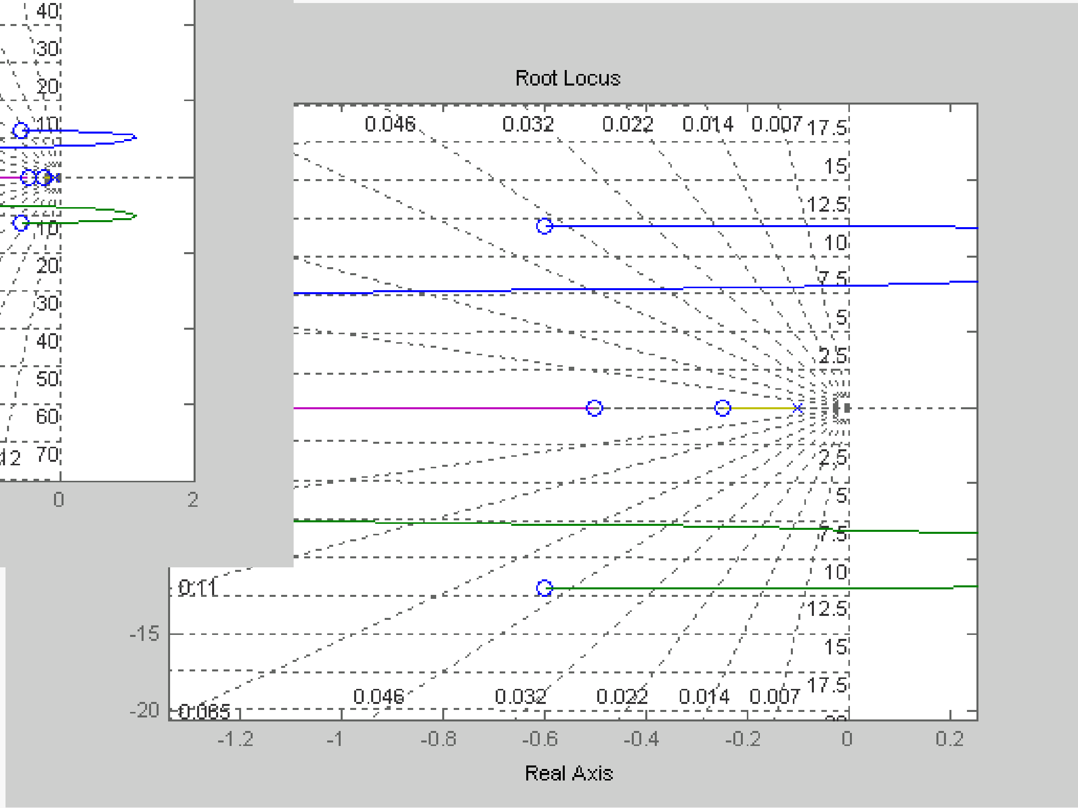
Uncompensated (P)

```
>> s=tf('s');
>> Gp=15*(25/(s+5)^2)*((s+0.5)/(s+0.1))*((s+0.25)/(s+4))*((s^2+2*.05*12*s+144)/(s^2+2*.25*8*s+64));
>> rlocus(Gp)
>> sgrid
>> [K,Poles]=rlocfind(G)
Select a point in the graphics window
sgrid
selected_point =
  -1.1137 + 7.2050i
K =
    0.0324
Poles =
  -7.0808 + 5.5584i
  -7.0808 - 5.5584i
  -1.1219 + 7.5434i
  -1.1219 - 7.5434i
  -1.5793
  -0.1154
>> Gce=1/(1+K*Gp)
```

Uncompensated (P)

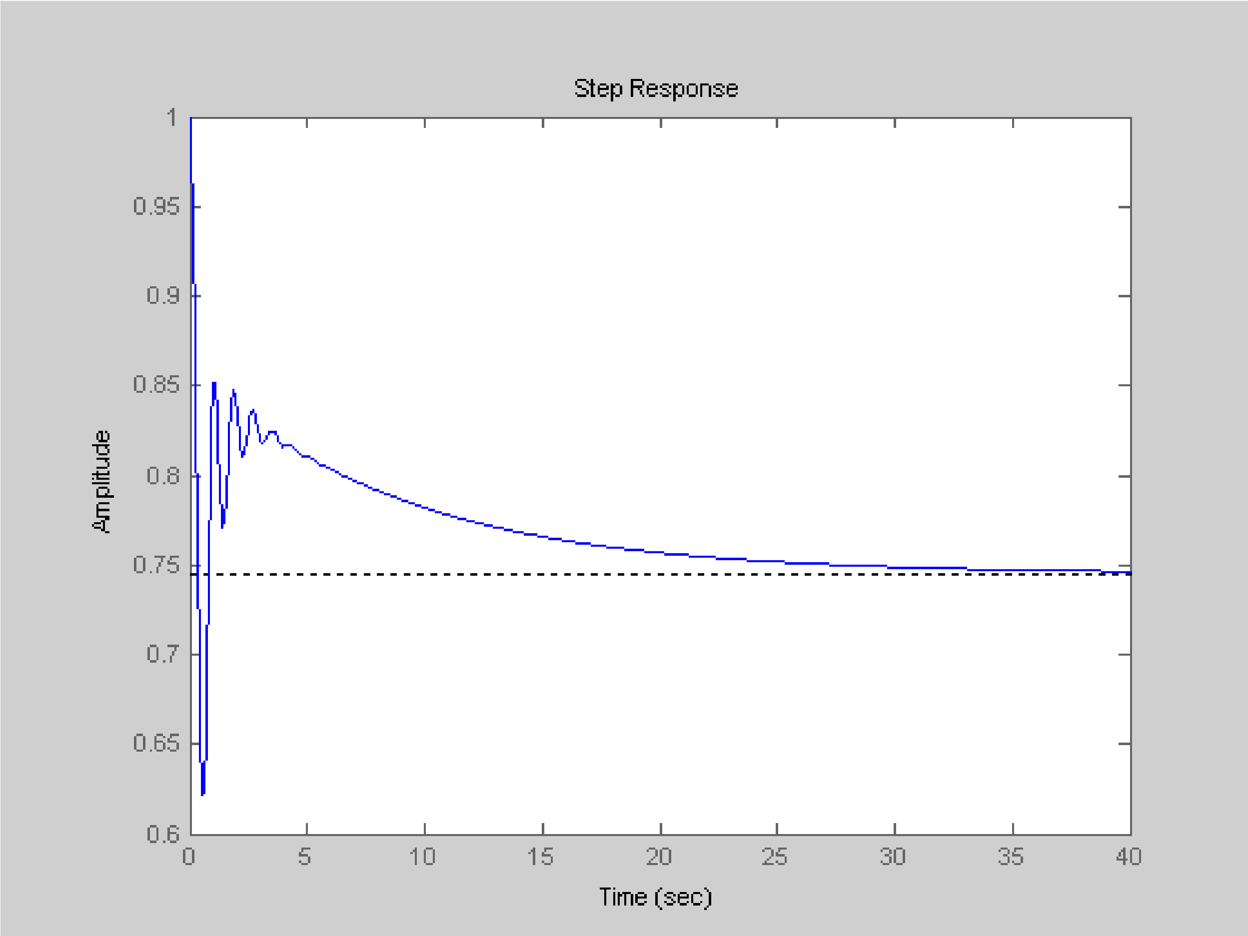
$$\lim_{s \to 0} sG_e(s)\frac{1}{s} = 0.7452$$





1st harmonic

Uncompensated (P), error response to step

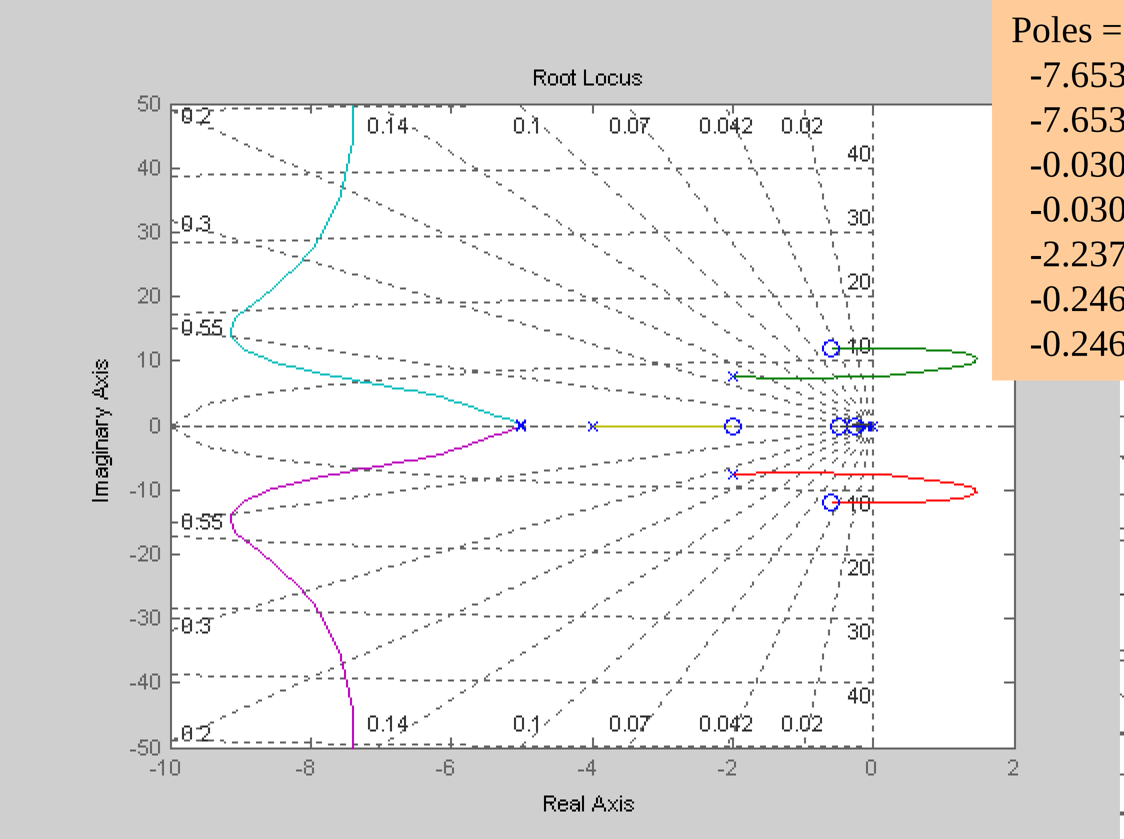


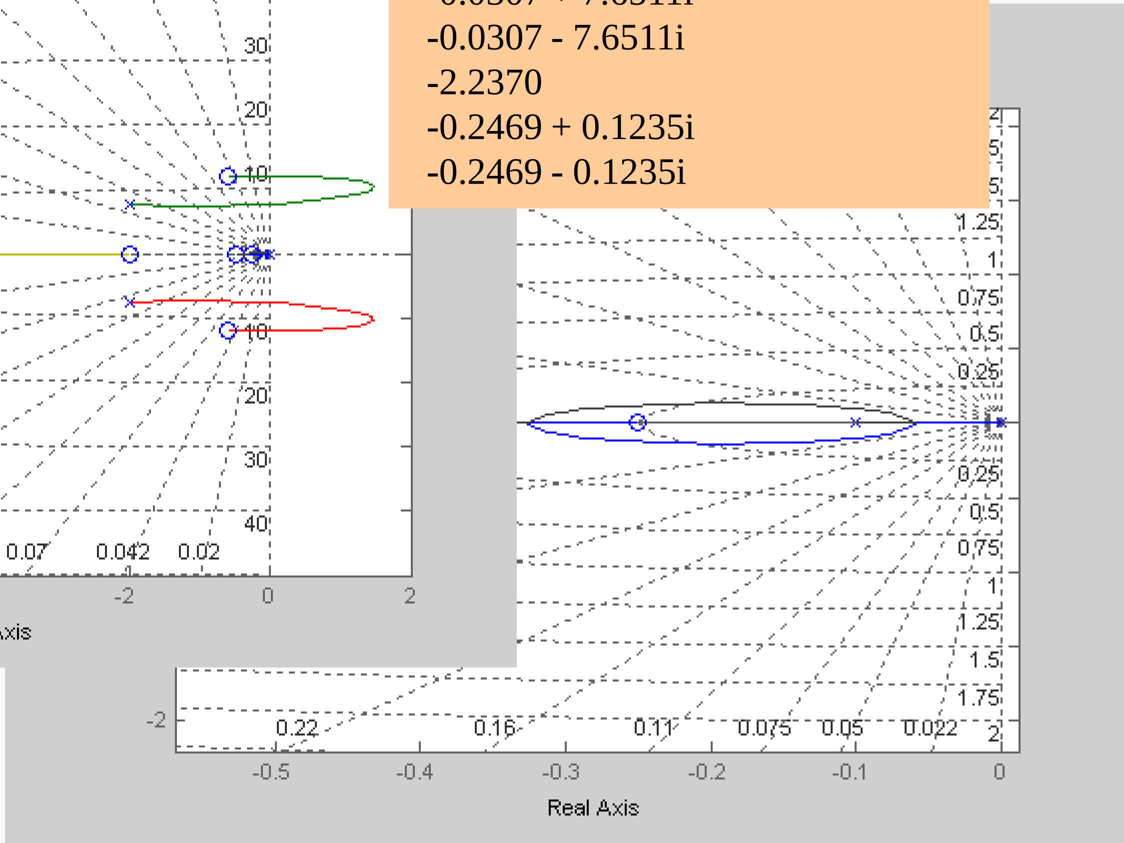
PI

$$G_c(s) = K\frac{s+2}{s}$$

Stability limit

```
>> [K,Poles]=rlocfind(G)
Select a point in the graphics window
selected_point =
  -0.0047 + 7.6087i
K =
    0.0861
Poles =
  -7.6539 + 7.6078i
  -7.6539 - 7.6078i
  -0.0307 + 7.6511i
  -0.0307 - 7.6511i
  -2.2370
  -0.2469 + 0.1235i
  -0.2469 - 0.1235i
```





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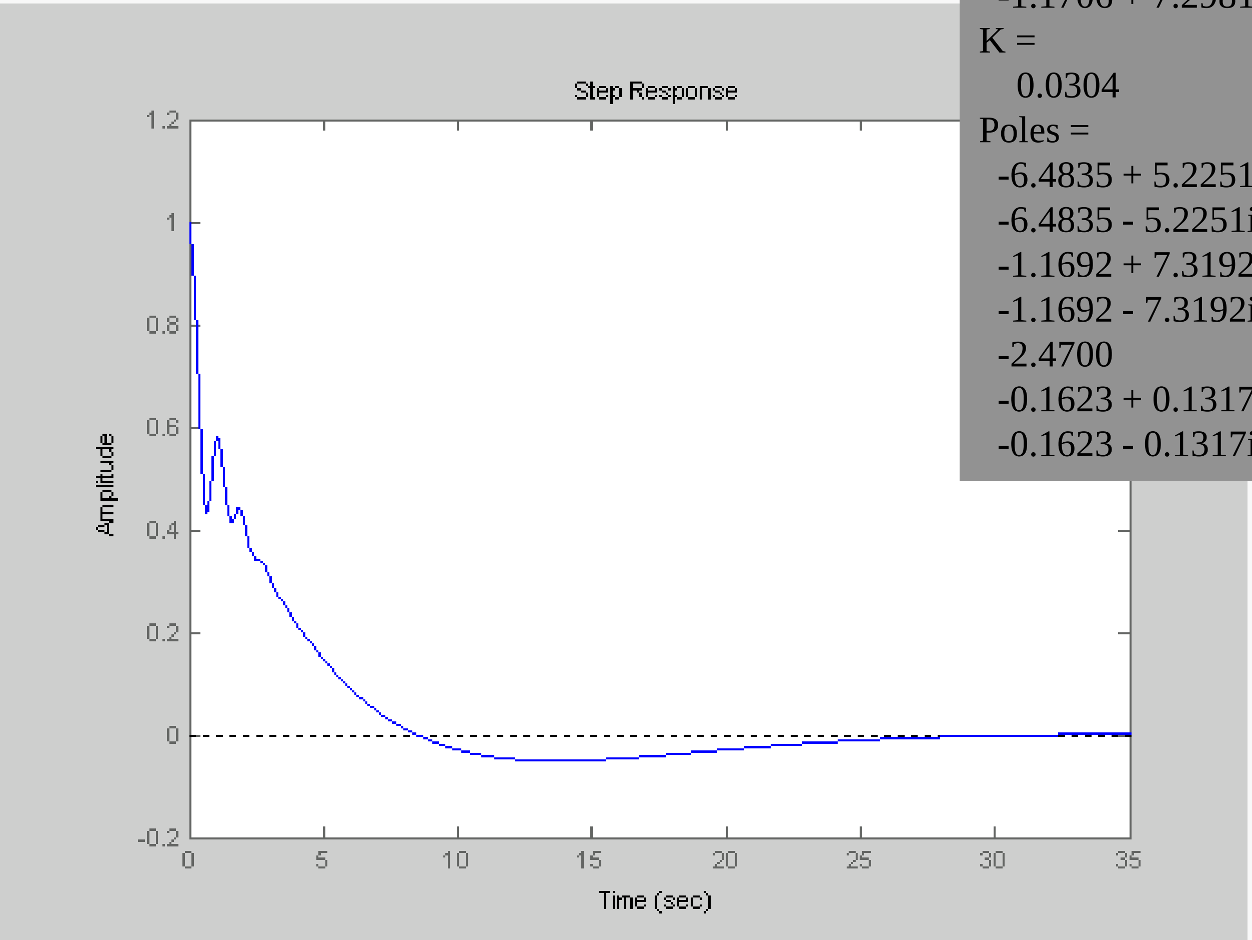
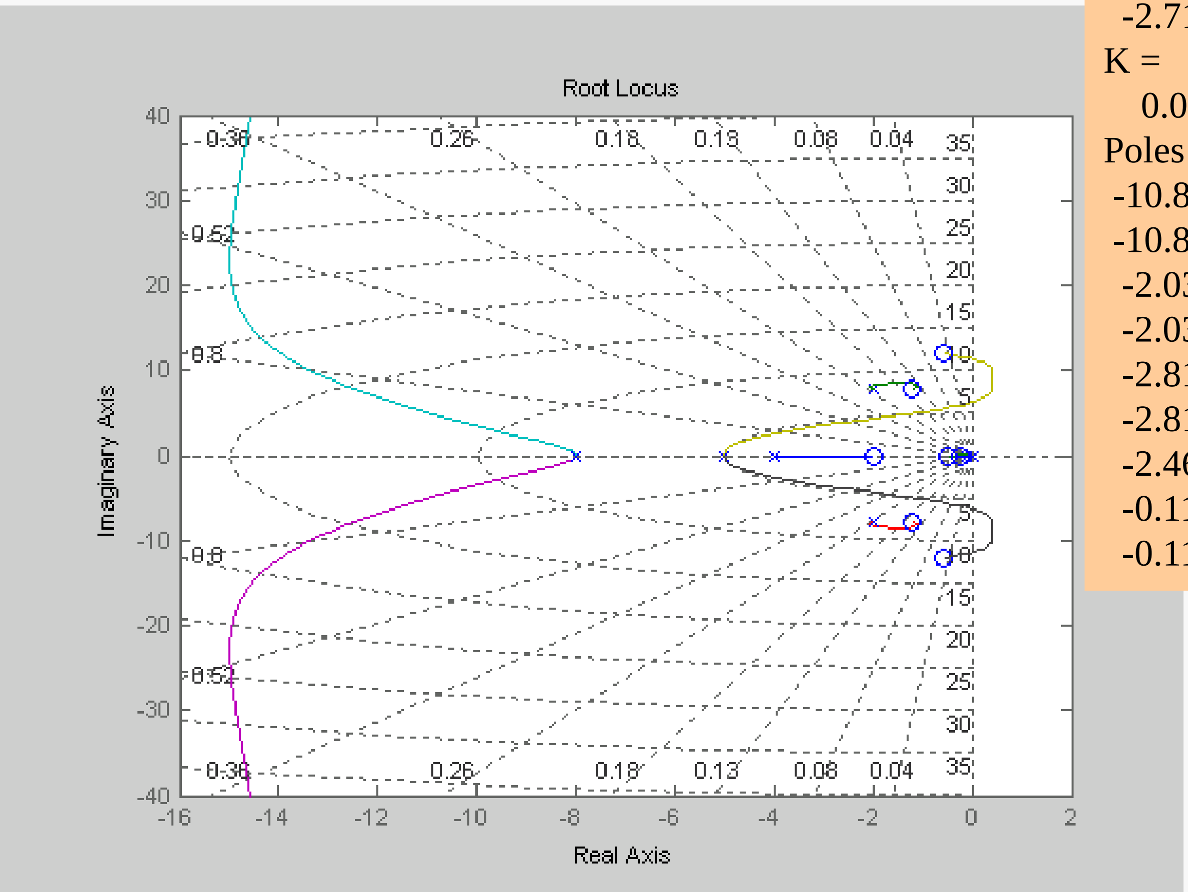
PI




```
>> [K,Poles]=rlocfind(G)
Select a point in the graphics window
selected_point =
  -1.1706 + 7.2981i
K =
    0.0304
Poles =
  -6.4835 + 5.2251i
  -6.4835 - 5.2251i
  -1.1692 + 7.3192i
  -1.1692 - 7.3192i
  -2.4700
  -0.1623 + 0.1317i
  -0.1623 - 0.1317i
```

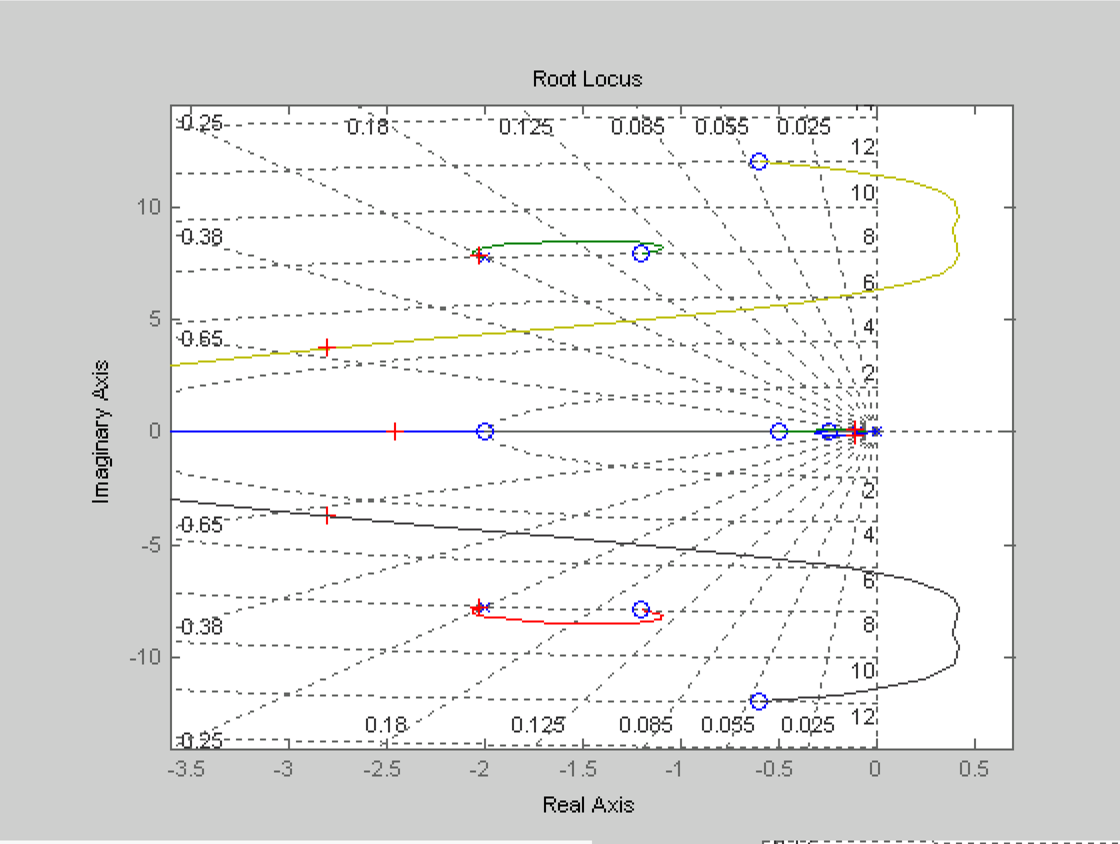
PI + Notch

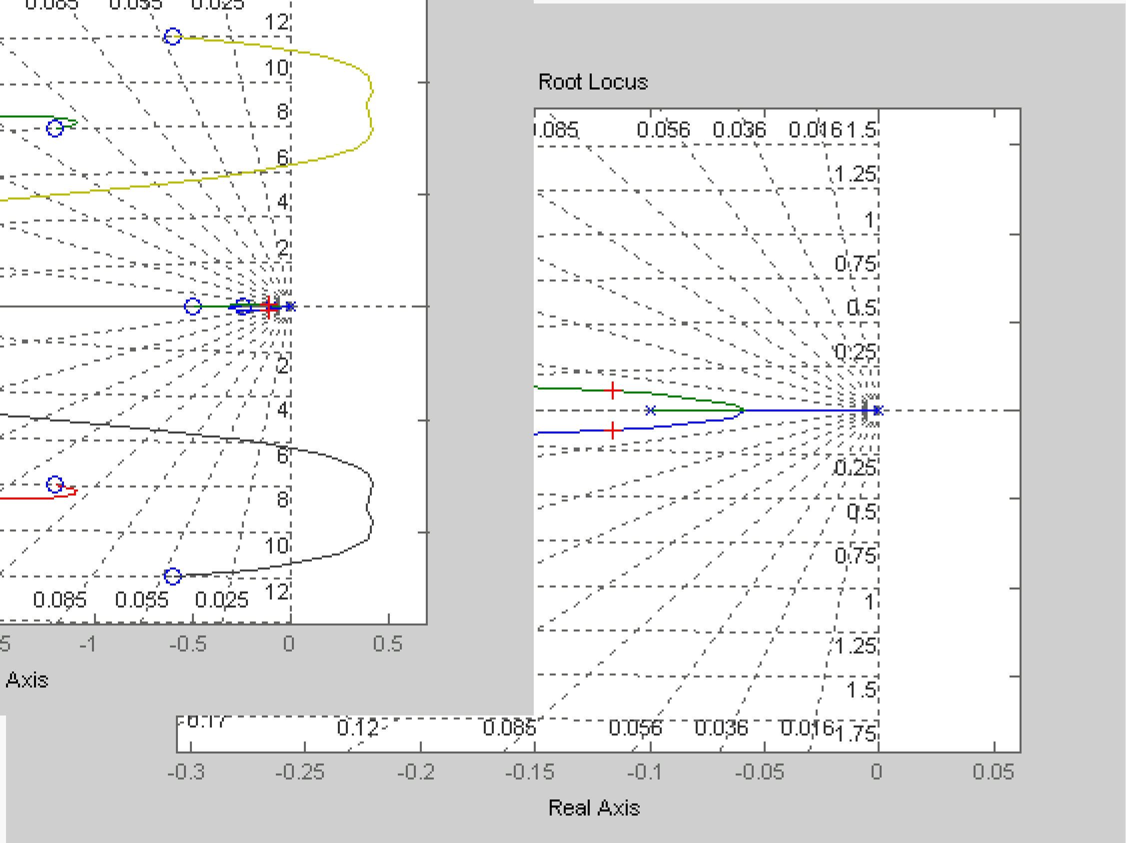
$$G_c(s) = K\frac{s+2}{s}\frac{s^2+2(0.15)8s+64}{(s+8)^2}$$



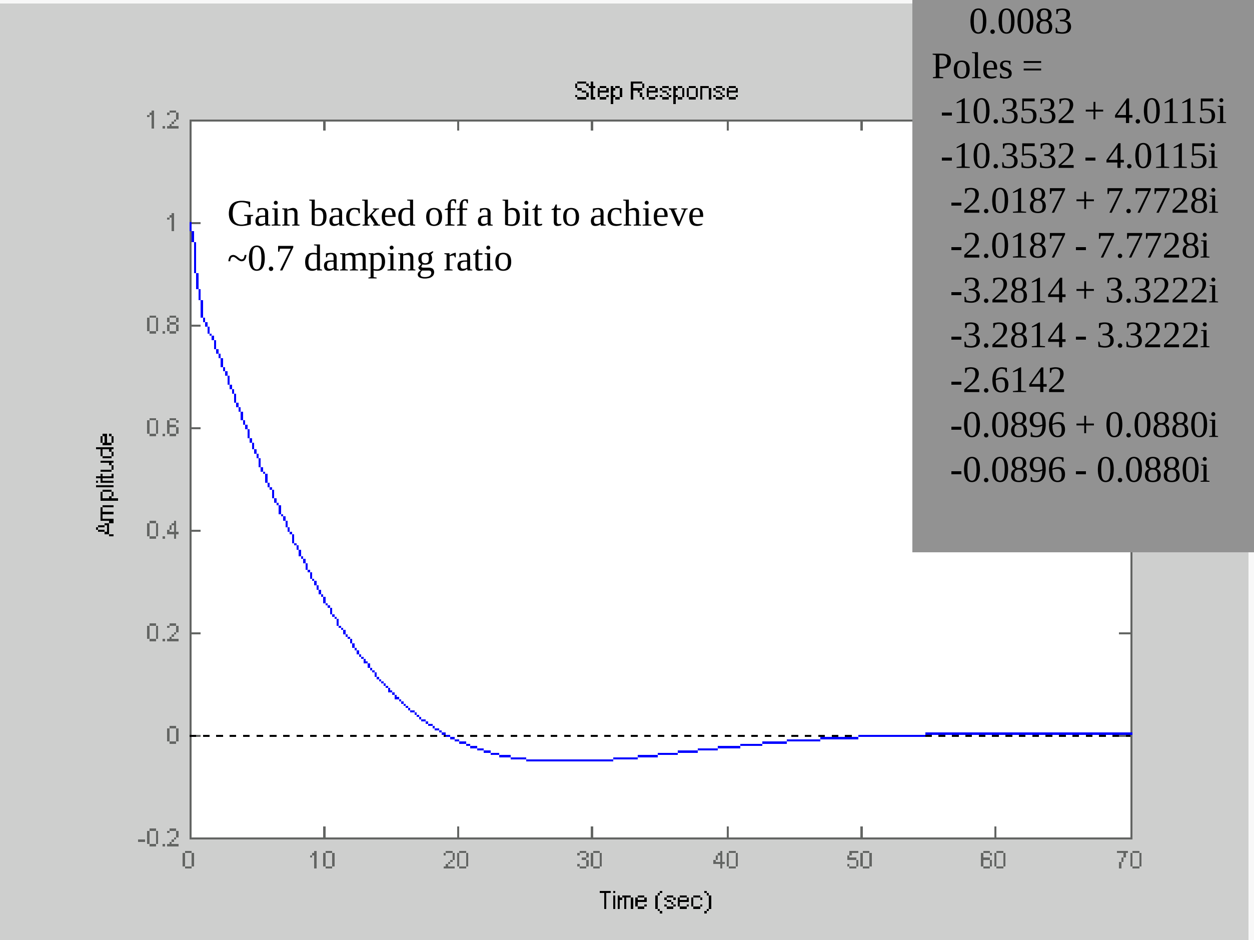

```
>> [K,Poles]=rlocfind(G)
Select a point in the graphics window
selected_point =
  -2.7133 + 3.6025i
K =
    0.0149
Poles =
 -10.8605 + 4.7906i
 -10.8605 - 4.7906i
  -2.0317 + 7.7960i
  -2.0317 - 7.7960i
  -2.8100 + 3.7251i
  -2.8100 - 3.7251i
  -2.4630
  -0.1163 + 0.1131i
  -0.1163 - 0.1131i
```

PI + Notch





PI + Notch




```
>> [K,Poles]=rlocfind(G)
Select a point in the graphics window
selected_point =
  -3.3104 + 3.3540i
K =
    0.0083
Poles =
 -10.3532 + 4.0115i
 -10.3532 - 4.0115i
  -2.0187 + 7.7728i
  -2.0187 - 7.7728i
  -3.2814 + 3.3222i
  -3.2814 - 3.3222i
  -2.6142
  -0.0896 + 0.0880i
  -0.0896 - 0.0880i
```

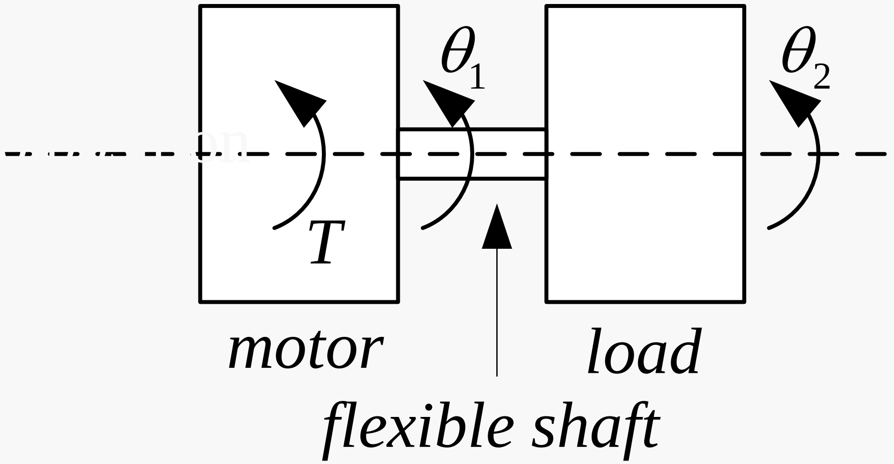
Hard Control Problems

Control design problems can be difficult for many reasons:

- Demanding specifications
- Nonlinearity
- Actuator/sensor constraints
- Coupling
- Implementation constraints

to name a few. But here are some deceptively simple issues involving linear SISO systems that can give a designer headaches.

Example: Collocated vs Non-collocated



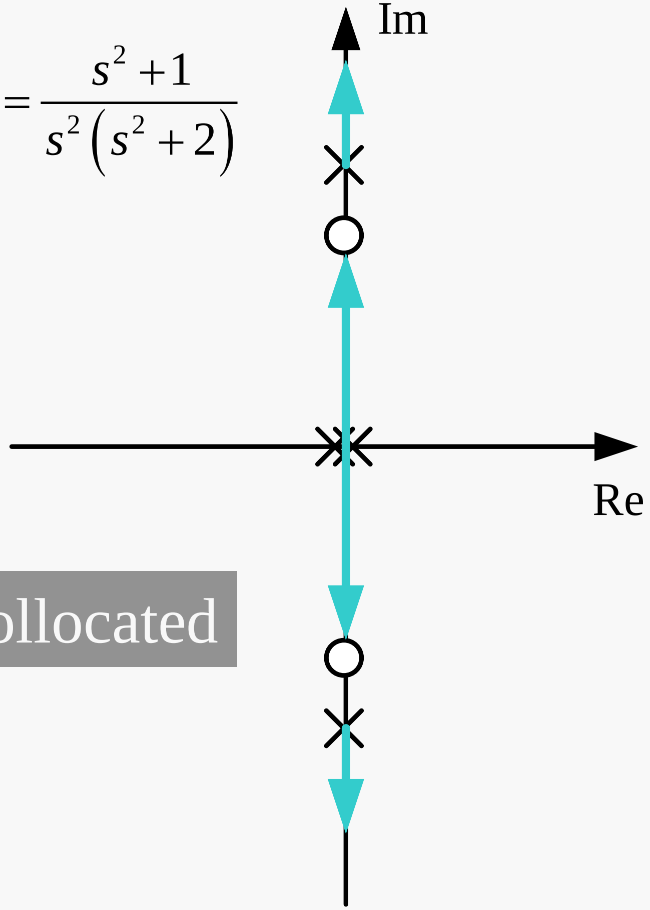
$$\Theta_1 = \frac{1}{J_1}\frac{s^2 + {k}/{J_2}}{s^2\left(s^2 + {(J_1+J_2)k}/{J_1 J_2}\right)}T$$

$$\Theta_2 = \frac{k}{J_1 J_2}\frac{1}{s^2\left(s^2 + {(J_1+J_2)k}/{J_1 J_2}\right)}T$$

$J_1 = J_2 = 1,\ k = 1$

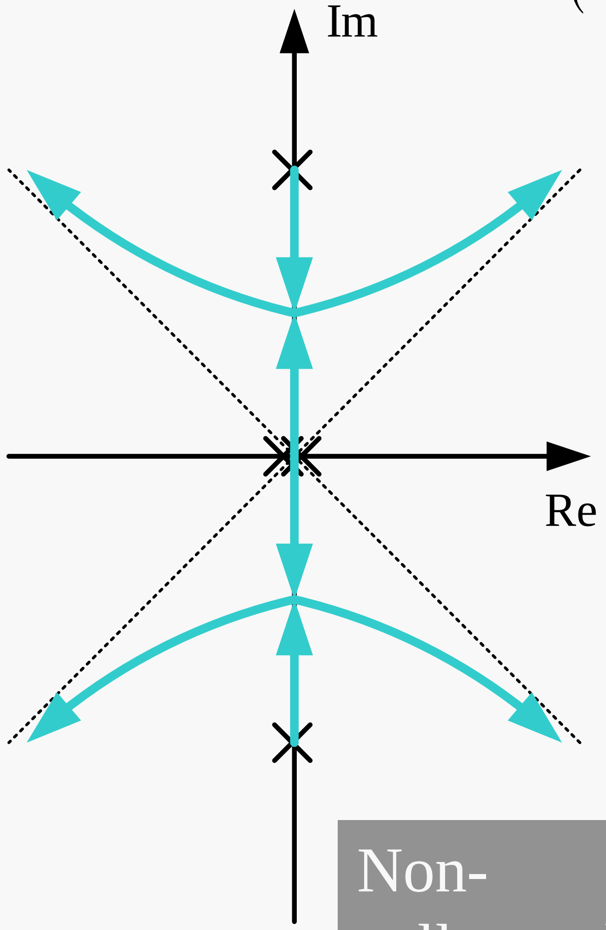
Example Cont’d

$$G_{\theta_1} = \frac{s^2+1}{s^2\left(s^2+2\right)}$$



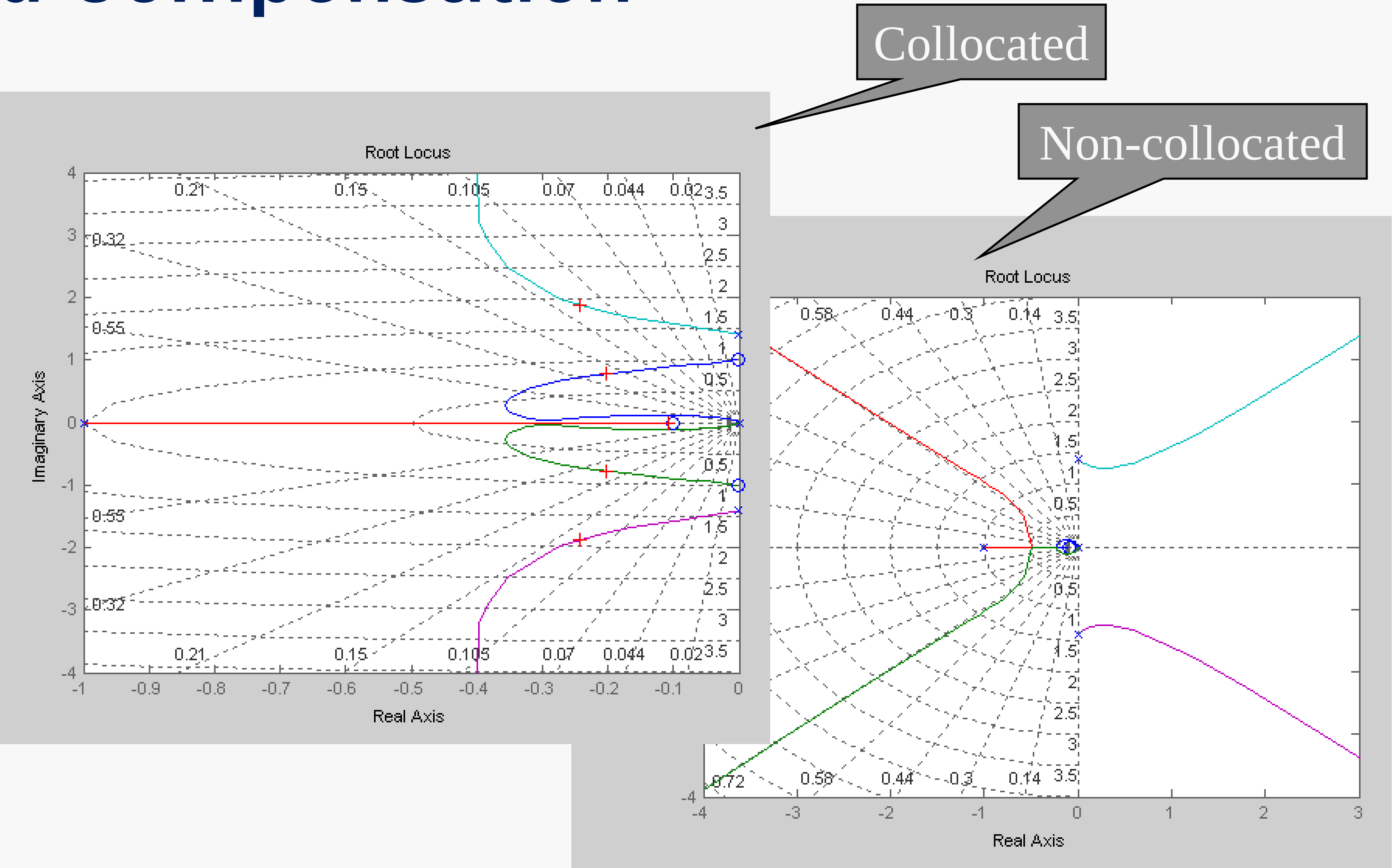
Collocated

$$G_{\theta_2} = \frac{1}{s^2\left(s^2+2\right)}$$

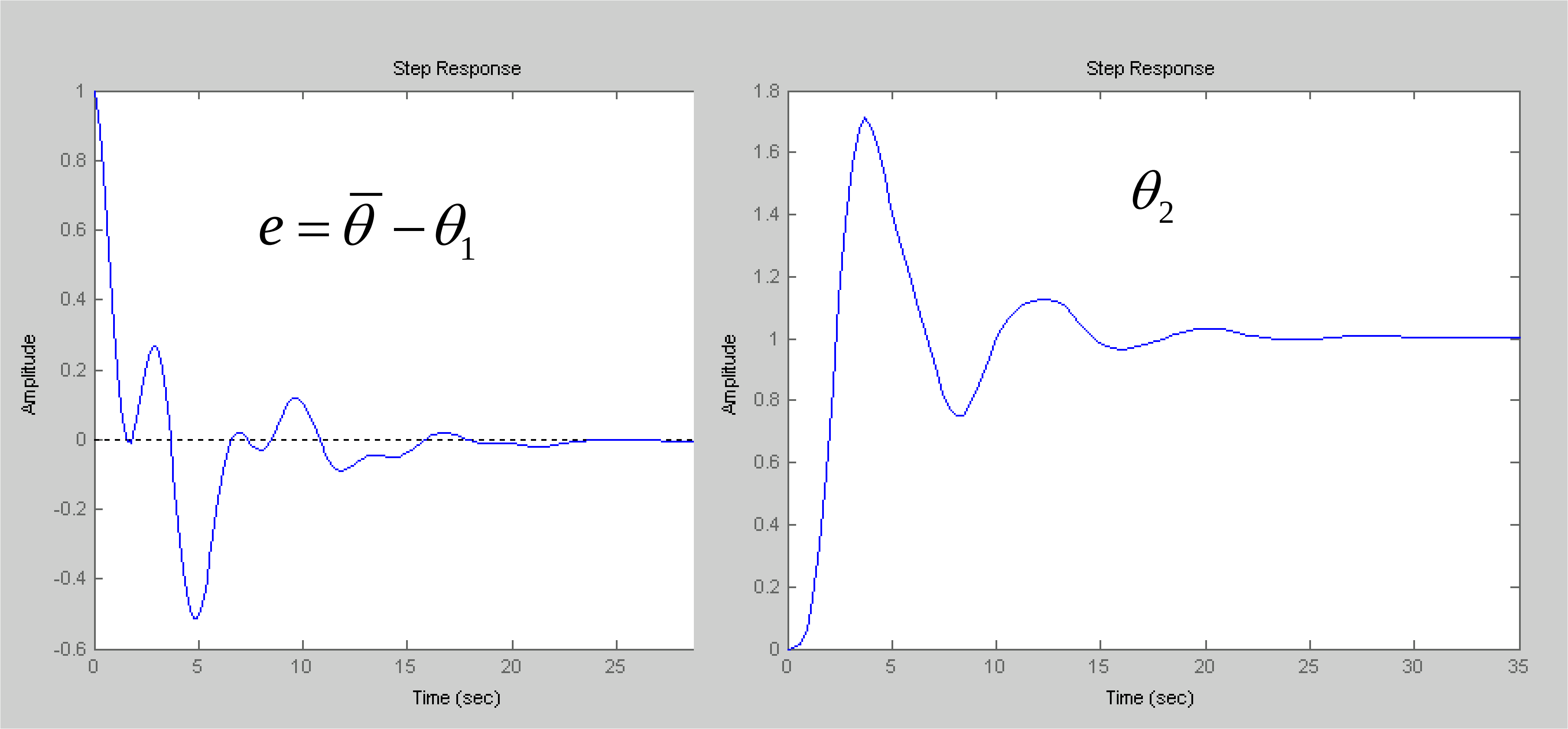


Non-collocated

Lead Compensation

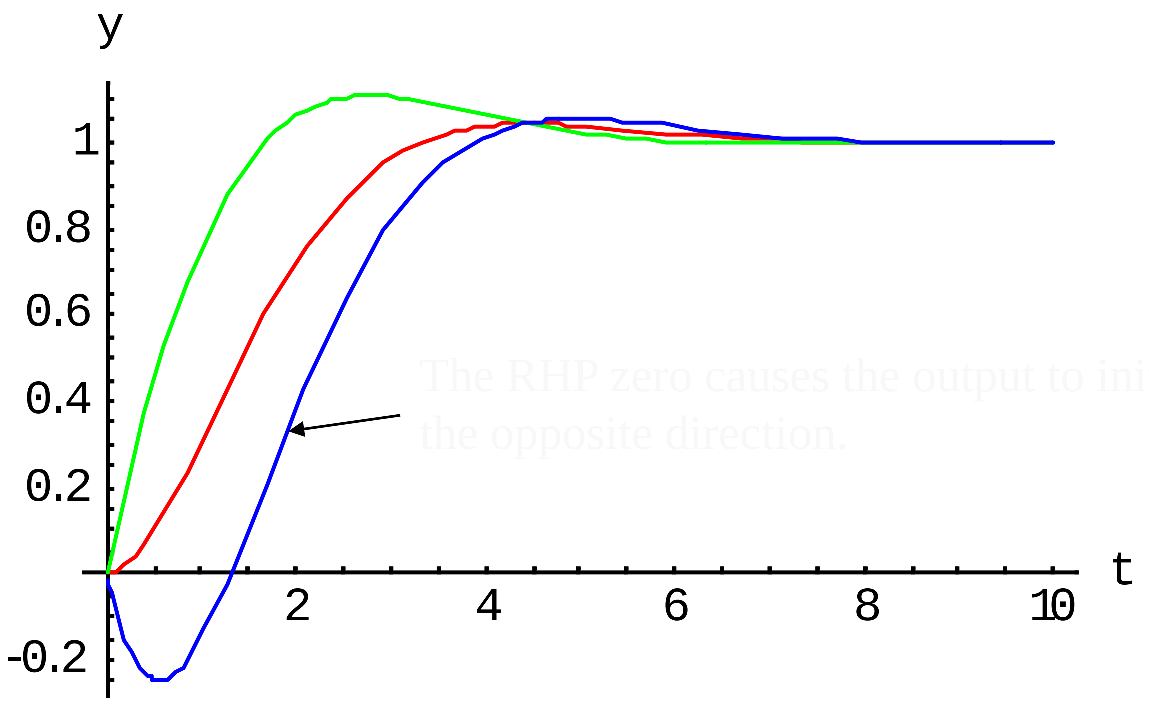


Lead Response (collocated case)



Right Half Plane Zero

$$G(s)=\frac{1}{s^2+\sqrt{2}s+1} \qquad G(s)=\frac{s+1}{s^2+\sqrt{2}s+1} \qquad G(s)=-\frac{s-1}{s^2+\sqrt{2}s+1}$$



Example: Simple Nonminimum Phase System

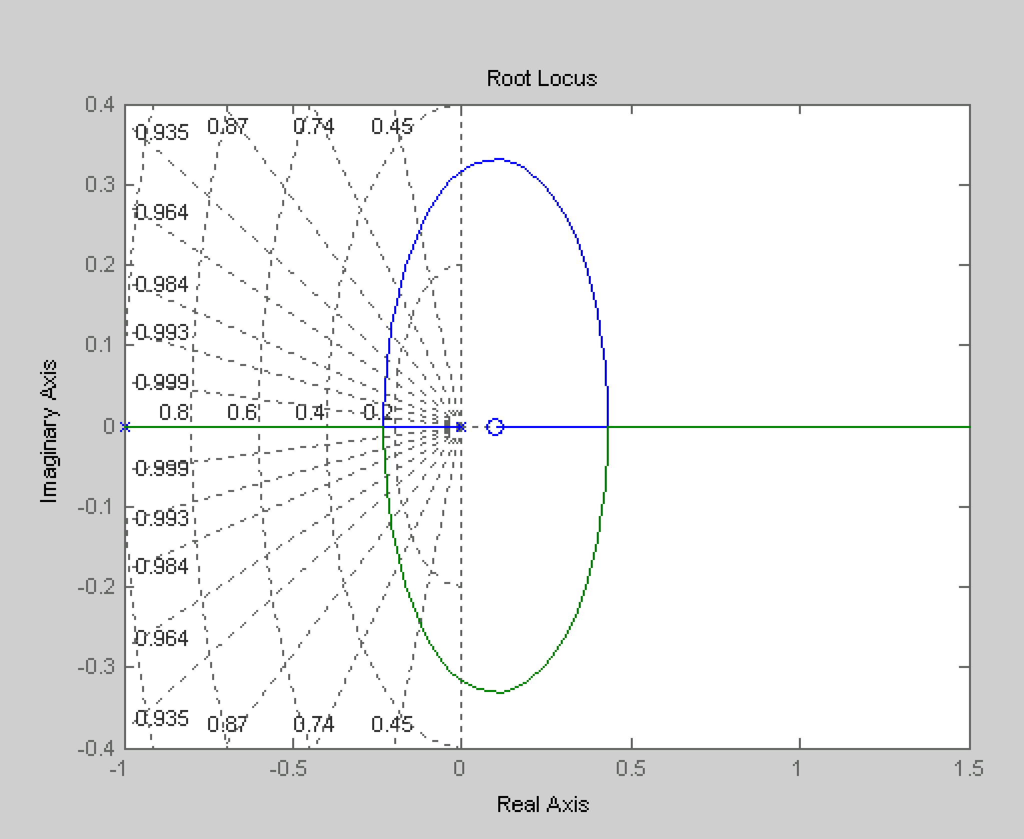
Consider the process

$$G_p(s) = \frac{-10s+1}{s(s+1)}$$

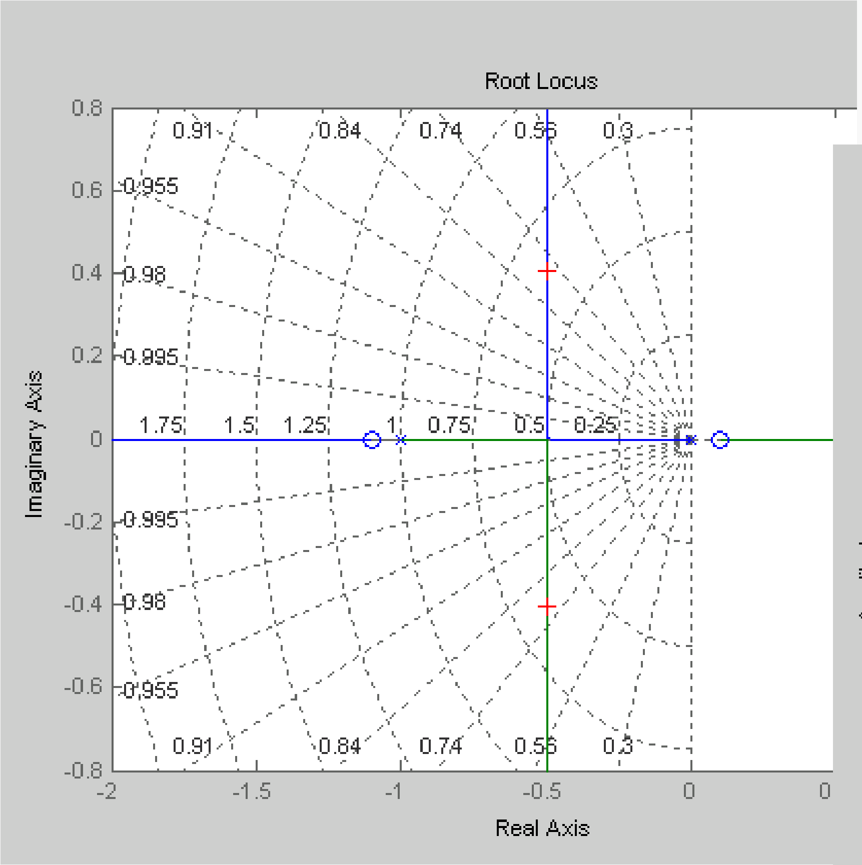
Try a PD compensator

$$G_c(s) = K(s+1.1)$$

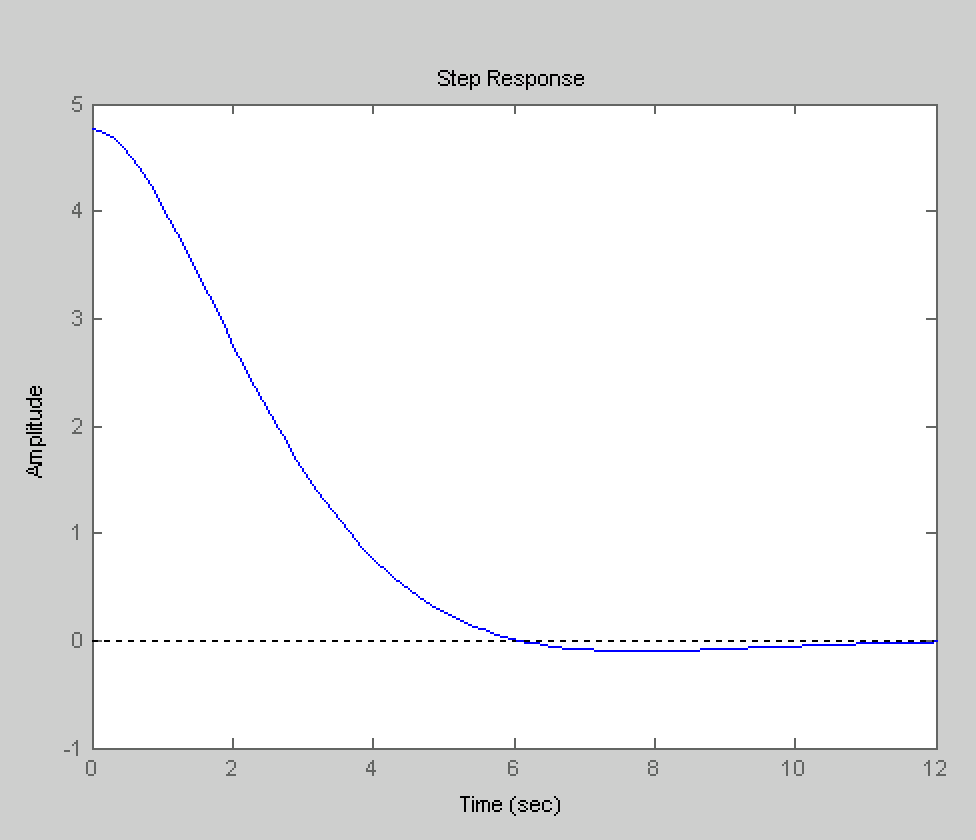
Uncompensated



Example with PD



Poles look decent, but look at the initial error.



Example: Check Initial Error

$$G_e = \frac{1}{1+G} \Rightarrow \frac{1}{1+K\left(s+1.1\right)\dfrac{-10s+1}{s\left(s+1\right)}}$$

$$G_e = \frac{s\left(s+1\right)}{\left(s^2+s\right)+K\left(s+1.1\right)\left(-10s+1\right)} = \frac{s\left(s+1\right)}{\left(1-10K\right)s^2+\left(1-10K\right)s+1.1K}$$

$$K = 0.0790 \text{ (from MATLAB)}$$

$$G_e = \frac{s\left(s+1\right)}{0.2099s^2+0.2099s+0.08691}$$

$$e\left(0^+\right) = \lim_{s\to\infty} sG_e\left(s\right)\frac{1}{s} \to \frac{1}{0.2099} = 4.7642$$

Initial Value Theorem

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F-16, X-29

$$G_p(s) = 1.645\frac{s(s+0.0423101)(s+0.586543)}{(s-0.730937)(s+1.7036)(s^2+0.0876334s+0.044546)}$$

Unstable Pole

$$G_p(s) = G(s)\frac{s-26}{s-6}$$

RHP pole-zero pair

Minimum phase part

Comments on Positive Feedback Root Locus

Positive feedback (or $K<0$) root locus rules are slightly different from the negative feedback rules. Here are the two changes.

4. Real-axis segments: For $K < 0$, real axis segments to the left of an even number of finite real axis poles and/or zeros are part of the root locus.

5. Behavior at infinity: The root locus approaches infinity along asymptotes with angles:

$$\theta = \frac{2k\pi}{\#\ finite\ poles - \#\ finite\ zeros},\ k = 0, \pm 1, \pm 2, \pm 3, \ldots$$

Furthermore, these asymptotes intersect the real axis at a common point given by

$$\sigma = \frac{\sum finite\ poles - \sum finite\ zeros}{\#\ finite\ poles - \#\ finite\ zeros}$$

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